

OLIMPIADA DE MATEMATICĂ

ETAPA LOCALĂ

8 februarie 2020

BAREM DE NOTARE

CLASA A VIII-A

1.)	Din oficiu	1p
	$36^n + 11 \cdot 6^n + 28 =$ $= (6^n)^2 + 4 \cdot 6^n + 7 \cdot 6^n + 4 \cdot 7 = 6^n \cdot (6^n + 4) + 7 \cdot (6^n + 4) = (6^n + 4)(6^n + 7)$	4p
	$5 \cdot 6^n + 35 = 5 \cdot (6^n + 7)$	1p
	$\frac{36^n + 11 \cdot 6^n + 28}{5 \cdot 6^n + 35} = \frac{6^n + 4}{5}$	1p
	Pentru orice $n \in \mathbb{N}^*$: ultima cifră a lui 6^n este 6, astfel ultima cifră a sumei $6^n + 4$ este 0, așadar $(6^n + 4):5$, deci $\frac{6^n + 4}{5} \in \mathbb{N}$.	2p
	Pentru $n = 0$, $\frac{6^n + 4}{5} = \frac{6^0 + 4}{5} = \frac{1 + 4}{5} = 1 \in \mathbb{N}$.	1p

2.)	Din oficiu	1p
	$a = \frac{\sqrt{2} - \sqrt{1}}{\sqrt{1} \cdot \sqrt{2}} + \frac{\sqrt{3} - \sqrt{2}}{\sqrt{2} \cdot \sqrt{3}} + \frac{\sqrt{4} - \sqrt{3}}{\sqrt{3} \cdot \sqrt{4}} + \frac{\sqrt{5} - \sqrt{4}}{\sqrt{4} \cdot \sqrt{5}} + \dots + \frac{\sqrt{25} - \sqrt{24}}{\sqrt{24} \cdot \sqrt{25}}$	2p
	$a = \frac{1}{\sqrt{1}} - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \frac{1}{\sqrt{4}} - \frac{1}{\sqrt{5}} + \dots + \frac{1}{\sqrt{24}} - \frac{1}{\sqrt{25}}$	1p
	$a = \frac{1}{1} - \frac{1}{5} = \frac{4}{5}$	1p
	$b = \frac{\sqrt{3} - 1}{2} + \frac{2 - \sqrt{2}}{2} + \frac{\sqrt{6} - 2}{2} + \frac{(1 + \sqrt{2} - \sqrt{3})^2}{4}$	1p
	$b = \frac{\sqrt{3} - 1 + 2 - \sqrt{2} + \sqrt{6} - 2}{2} + \frac{6 + 2\sqrt{2} - 2\sqrt{3} - 2\sqrt{6}}{4}$	2p
	$b = \frac{2\sqrt{3} - 2 - 2\sqrt{2} + 2\sqrt{6} + 6 + 2\sqrt{2} - 2\sqrt{3} - 2\sqrt{6}}{4}$	
	$b = \frac{4}{4} = 1$	1p
	$(b - a)^{-1} = \left(1 - \frac{4}{5}\right)^{-1} = \left(\frac{1}{5}\right)^{-1} = 5 \in \mathbb{N}$	1p

3.)	Din oficiu	1p
		1p
a)	$\Delta BAC : m(\sphericalangle A) = 90^\circ, AD \text{ mediană} \Rightarrow AD = \frac{BC}{2} \Rightarrow AD = 10 \text{ (cm)}.$ $MA \perp (ABC) \Rightarrow pr_{(ABC)} MD = DA \Rightarrow \sphericalangle(MD, (ABC)) = \sphericalangle MDA$ $\Delta MAD : m(\sphericalangle A) = 90^\circ \Rightarrow tg(\sphericalangle MDA) = \frac{MA}{DA} = \frac{12}{10} = \frac{6}{5}$	2p 1p 1p
b)	$\left. \begin{matrix} MA \perp (ABC) \\ AE \perp BC \end{matrix} \right\} \xrightarrow{T.3P} ME \perp BC \Rightarrow A_{\Delta MBC} = \frac{BC \cdot ME}{2}$	1p
	$\Delta ADC : AD = DC \Rightarrow m(\sphericalangle DAC) = m(\sphericalangle ACD) = 15^\circ$ $m(\sphericalangle ADE) = 30^\circ \text{ (unghi exterior)} \Rightarrow \Delta AED : AE = \frac{AD}{2} = 5 \text{ cm}.$	2p
	$\Delta MAE : m(\sphericalangle A) = 90^\circ \xrightarrow{t.Pit.} ME = 13 \text{ cm} \Rightarrow A_{\Delta MBC} = \frac{20 \cdot 13}{2} = 130 \text{ (cm}^2\text{)}.$	1p

4.)	Din oficiu	1p
		1p
	a) desen	1p
	$\left. \begin{matrix} [AM] \equiv [MB] \\ [BN] \equiv [NC] \end{matrix} \right\} \Rightarrow MN \parallel AC (l.m) \left. \begin{matrix} A'C'CA \text{ dreptunghi} \\ \Rightarrow A'C' \parallel AC \end{matrix} \right\} \Rightarrow A'C' \parallel MN$ $\left. \begin{matrix} A'C' \parallel MN \\ MN \subset (D'MN) \end{matrix} \right\} \Rightarrow A'C' \parallel (D'MN)$	1p 1p
	b) $MN \cap DB = \{S\}, AC \cap DB = \{O\}$ $\left. \begin{matrix} D'D \perp (ABC) \\ DS \perp AC \\ AC \parallel MN \end{matrix} \right\} \xrightarrow{T.3P} DS \perp MN \Rightarrow D'S \perp MN$	1p

	$\left. \begin{array}{l} (D'MN) \cap (ABC) = MN \\ D'S \perp MN, D'S \subset (D'MN) \\ DS \perp MN, DS \subset (ABC) \end{array} \right\} \Rightarrow \sphericalangle((D'MN), (ABC)) = \sphericalangle D'SD$	1p
	$DS = DO + OS = \frac{DB}{2} + \frac{DB}{4} = \frac{3}{4}DB = \frac{3}{4} \cdot AB\sqrt{2} = 3\sqrt{2} \text{ (cm)}$	0,5 p
	$\Delta D'DS : m(\sphericalangle D) = 90^\circ \Rightarrow \operatorname{tg}(\sphericalangle D'SD) = \frac{D'D}{DS} = \frac{3\sqrt{6}}{3\sqrt{2}} = \sqrt{3} \Rightarrow m(\sphericalangle D'SD) = 60^\circ$	0,5 p
c)	$\left. \begin{array}{l} MN \perp D'S \\ MN \perp DS \end{array} \right\} \Rightarrow MN \perp (D'DS) \left\{ \begin{array}{l} \Rightarrow (D'DS) \perp (D'MN) \\ MN \subset (D'MN) \end{array} \right.$	1p
	$\left. \begin{array}{l} (D'DS) \cap (D'MN) = D'S \\ DT \perp D'S \end{array} \right\} \Rightarrow d(D, (D'MN)) = DT$	1p
	$\Delta D'DS : m(\sphericalangle D) = 90^\circ \xrightarrow{t.Pit.} D'S = 6\sqrt{2}$ $DT \perp D'S \Rightarrow DT = \frac{D'D \cdot DS}{D'S} = \frac{3\sqrt{6} \cdot 3\sqrt{2}}{6\sqrt{2}} = \frac{3\sqrt{6}}{2}$	0,5 p 0,5 p