OLIMPIADA DE MATEMATICĂ

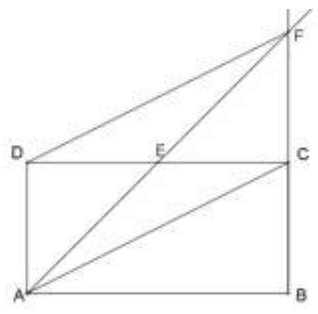
ETAPA LOCALĂ

8 februarie 2020

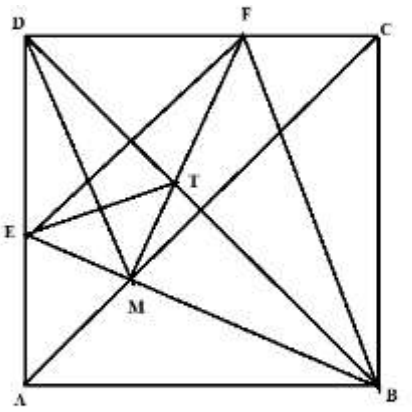
BAREM DE NOTARE
CLASA A VII-A

1.)	**Din oficiu**	**1p**
	$A=\sqrt{2}+\sqrt{2\cdot 2^2}+\sqrt{2\cdot 3^2}+\sqrt{2\cdot 4^2}+\sqrt{2\cdot 5^2}+...+\sqrt{2\cdot 31^2}=$ $=\sqrt{2}+2\sqrt{2}+3\sqrt{2}+4\sqrt{2}+5\sqrt{2}+...+31\sqrt{2}=$ $=(1+2+3+4+5+...+31)\sqrt{2}=\frac{31\cdot 32}{2}\sqrt{2}=496\sqrt{2}$	**1p** **1p** **1p**
	$B=\sqrt{2^{12}\cdot(1+2^3)}\cdot\frac{9}{\sqrt{6}}=\sqrt{2^{12}\cdot 9}\cdot\frac{9\sqrt{6}}{6}=2^6\cdot 3\cdot\frac{3\sqrt{6}}{2}=2^5\cdot 9\sqrt{6}=288\sqrt{6}$	**3p**
	$A=31\cdot 16\sqrt{2}$, $B=16\sqrt{2}\cdot 18\sqrt{3}$ $31=\sqrt{961}$, $18\sqrt{3}=\sqrt{972}\Rightarrow 31<18\sqrt{3}\Rightarrow A<B$	**1p** **2p**

2.)	**Din oficiu**	**1p**
	$a=\frac{1}{7}+\frac{9}{14}+\frac{10}{21}+\frac{11}{28}+...+\frac{70}{441}-\left(\frac{7}{14}+\frac{7}{21}+\frac{7}{28}+...+\frac{7}{441}\right)$	**2p**
	$a=\frac{1}{7}+\frac{9-7}{14}+\frac{10-7}{21}+\frac{11-7}{28}+...+\frac{70-7}{441}=\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+...+\frac{1}{7}=\frac{63}{7}=9=3^2$	**3p**
	$1+3+5+...+2021=\frac{(1+2021)\cdot 1011}{2}=\frac{2022\cdot 1011}{2}=1011\cdot 1011=1011^2$	**2p**
	$b=1+3+5+...+1011=\frac{(1+1011)\cdot 506}{2}=\frac{1012\cdot 506}{2}=506\cdot 506=506^2$	**2p**

3.)	**Din oficiu**	**1p**
a).	 $\left.\begin{array}{l} DE\equiv EC\,(Ip.)\\ DEA\sphericalangle\equiv CEF\sphericalangle\,(\sphericalangle op.la\,vârf)\\ EDA\sphericalangle\equiv ECF\sphericalangle\,(90^\circ)\end{array}\right\}\Rightarrow \Delta DEA\equiv\Delta CEF\ (1)$	**2p**
	$(1)\Rightarrow AE\equiv EF$	**1p**
	$\left.\begin{array}{l} AE\equiv EF\\ DE\equiv EC\end{array}\right\}\Rightarrow ACFD$ paralelogram (2)	**1p**
	$(2)\Rightarrow DF\parallel AC$	**1p**

b).	Notăm $A_{AEC} = a$ $\left.\begin{array}{l} A_{AEC} = \dfrac{EC \cdot AD}{2} \\ A_{AED} = \dfrac{ED \cdot AD}{2} \\ EC \equiv ED \end{array}\right\} \Rightarrow A_{AED} = A_{AEC} = a$	**1p**
	$\Delta DAC \equiv \Delta BCA \equiv \Delta CFD (1)$	**1p**
	$(1) \Rightarrow A_{DAC} = A_{BCA} = A_{CFD} = 2a$	**1p**
	$A_{ABFD} = 6a = 6 \cdot A_{AEC} \Leftrightarrow A_{AEC} = \dfrac{1}{6} \cdot A_{ABFD}$	**1p**

4.)	**Din oficiu**	**1p**
	 **a)** În ΔADB, BE și AC sunt bisectoare, deci M este intersecția bisectoarelor $\Rightarrow DM$ bisectoare.	**1p**
	$\sphericalangle BMT = \sphericalangle BAE$ *și* $\sphericalangle TBM = \sphericalangle EBA \Rightarrow \sphericalangle MTB = \sphericalangle AEB$	**1p**
	$\sphericalangle MED = 180^0 - \sphericalangle AEB = 180^0 - \sphericalangle MTB = \sphericalangle MTD \Rightarrow \sphericalangle TMD = \sphericalangle EMD \Rightarrow \Delta DMT \equiv \Delta DME$ *(U.L.U.)* $\Rightarrow DT \equiv DE$, deci ΔDET isoscel	**2p**
	b) *MEDF* este patrulater convex, deci $\sphericalangle MED + \sphericalangle MFD = 180^0 \Rightarrow \sphericalangle MFD = \sphericalangle AEB = \sphericalangle BTM = \sphericalangle DTF$, deci ΔDFT isoscel.	**1p**
	$DT = DE = DF \Rightarrow \Delta DEF$ dreptunghic isoscel $\Rightarrow \sphericalangle DEF = 45^0 = \sphericalangle DAC$ ($\sphericalangle$corespondente) $\Rightarrow EF \parallel AC$	**2p**
	c) $BT \perp AC \Rightarrow BT \perp EF$, $FT \perp EB \Rightarrow T$ ortocentru în $\Delta BEF \Rightarrow ET$ înălțime $\Rightarrow ET \perp BF$.	**2p**